

Problem 1.11

[Difficulty: 3]

1.11 For a small particle of styrofoam (1 lbf/ft^3) (spherical, with diameter $d = 0.3 \text{ mm}$) falling in standard air at speed V , the drag is given by $F_D = 3\pi\mu Vd$, where μ is the air viscosity. Find the maximum speed starting from rest, and the time it takes to reach 95 percent of this speed. Plot the speed as a function of time.

Given: Data on sphere and formula for drag.

Find: Maximum speed, time to reach 95% of this speed, and plot speed as a function of time.

Solution: Use given data and data in Appendices, and integrate equation of motion by separating variables.

The data provided, or available in the Appendices, are:

$$\rho_{\text{air}} = 1.17 \cdot \frac{\text{kg}}{\text{m}^3} \quad \mu = 1.8 \times 10^{-5} \cdot \frac{\text{N}\cdot\text{s}}{\text{m}^2} \quad \rho_w = 999 \cdot \frac{\text{kg}}{\text{m}^3} \quad \text{SG}_{\text{Sty}} = 0.016 \quad d = 0.3 \cdot \text{mm}$$

Then the density of the sphere is

$$\rho_{\text{Sty}} = \text{SG}_{\text{Sty}} \cdot \rho_w \quad \rho_{\text{Sty}} = 16 \cdot \frac{\text{kg}}{\text{m}^3}$$

The sphere mass is

$$M = \rho_{\text{Sty}} \cdot \frac{\pi \cdot d^3}{6} = 16 \cdot \frac{\text{kg}}{\text{m}^3} \times \pi \times \frac{(0.0003 \cdot \text{m})^3}{6} \quad M = 2.26 \times 10^{-10} \text{ kg}$$

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects)

$$M \cdot g = 3 \cdot \pi \cdot V \cdot d$$

so

$$V_{\text{max}} = \frac{M \cdot g}{3 \cdot \pi \cdot \mu \cdot d} = \frac{1}{3 \cdot \pi} \times 2.26 \times 10^{-10} \cdot \text{kg} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times \frac{\text{m}^2}{1.8 \times 10^{-5} \cdot \text{N}\cdot\text{s}} \times \frac{1}{0.0003 \cdot \text{m}} \quad V_{\text{max}} = 0.0435 \frac{\text{m}}{\text{s}}$$

Newton's 2nd law for the general motion is (ignoring buoyancy effects)

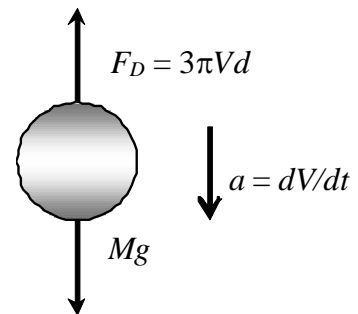
$$M \cdot \frac{dV}{dt} = M \cdot g - 3 \cdot \pi \cdot \mu \cdot V \cdot d$$

so

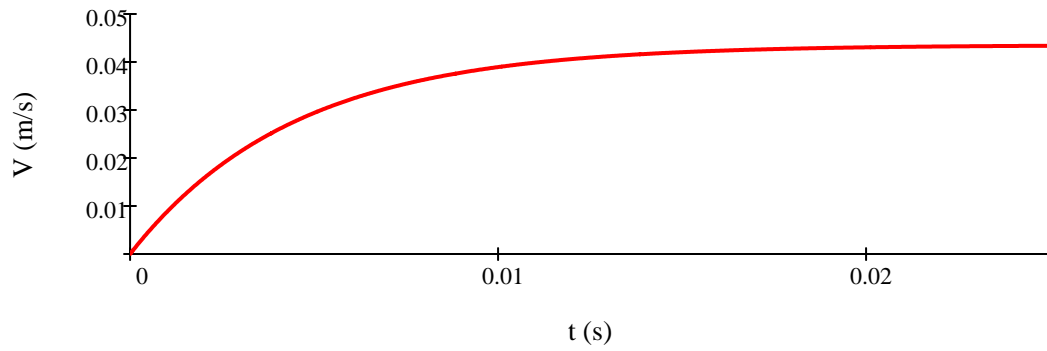
$$\frac{dV}{g - \frac{3 \cdot \pi \cdot \mu \cdot d}{M} \cdot V} = dt$$

Integrating and using limits

$$V(t) = \frac{M \cdot g}{3 \cdot \pi \cdot \mu \cdot d} \cdot \left(1 - e^{\frac{-3 \cdot \pi \cdot \mu \cdot d}{M} \cdot t} \right)$$



Using the given data



The time to reach 95% of maximum speed is obtained from

$$\frac{M \cdot g}{3 \cdot \pi \cdot \mu \cdot d} \cdot \left(1 - e^{\frac{-3 \cdot \pi \cdot \mu \cdot d}{M} \cdot t} \right) = 0.95 \cdot V_{\max}$$

so
$$t = -\frac{M}{3 \cdot \pi \cdot \mu \cdot d} \cdot \ln \left(1 - \frac{0.95 \cdot V_{\max} \cdot 3 \cdot \pi \cdot \mu \cdot d}{M \cdot g} \right)$$

Substituting values $t = 0.0133 \text{ s}$

The plot can also be done in *Excel*.